Calculus Warm Up #6-3

Same groups as yesterday.

Compare HW: p. 197 #9 and 19

HW Questions: p. 197

9. $y = 3x^4 + 4x^3$

19. $f(x) = \dfrac{x^2}{x^2 + 3}$

Places of interest and intervals between	test #'s on the interval	f′(x)	f″(x)	Conclusion

33. $f(x) = \dfrac{x^2 + 1}{x}$

Places of interest and intervals between	test #'s on the interval	f′(x) +, - or 0	f″(x) +, - or 0	Conclusion	
$(-\infty, -1)$		+	−	f incr	Concave down
$x = -1$	-2	0	−	Max	
$(-1, 0)$		−	−	f decr.	
$x = 0$	undef	undef	undef	vert asymptote	
$(0, 1)$		−	+	f decr	Concave up
$x = 1$	2	0	+	Min	
$(1, \infty)$		+	+	f incr	

p. 203

In Exercises 2–6, find two positive numbers that satisfy the given requirements.

5. The second number is the reciprocal of the first and their sum is minimum.

In Exercises 9 and 10, find the length and width of a rectangle of minimum perimeter for the given area.

9. Area: 64 square feet

In Exercises 11 and 12, find the point on the graph of the function closest to the given point.

Function	Point
11. $f(x) = \sqrt{x}$	(4, 0)

(x, y)

minimize distance

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

primary equation.

$y = \sqrt{x}$ is secondary relationship

$d = \sqrt{(x-4)^2 + (y-0)^2}$

$d = \sqrt{(x-4)^2 + (\sqrt{x})^2}$

$d = \sqrt{x^2 - 7x + 16}$

* Notice: minimum will be at smallest value under radical → let $g(x) = x^2 - 7x + 16$

$g'(x) = 2x - 7$

$0 = 2x - 7$

$x = \frac{7}{2}$

$\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

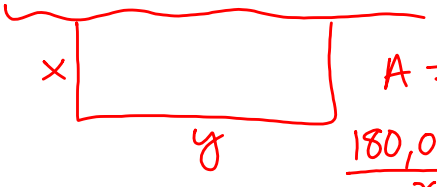
$\frac{\sqrt{14}}{2}$

confirm min

g': − on $(0, \frac{7}{2})$, + on $(\frac{7}{2}, 10)$

$g''(x) = 2$ always + conc up ∪

13. A dairy farmer plans to fence in a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?



$A = 180{,}000\ m^2$

$\frac{180{,}000}{x} = \frac{xy}{x}$

$F = 2x + y$

$F(x) = 2x + 180{,}000x^{-1}$

$F'(x) = 2 - \frac{180{,}000}{x^2}$

$0 =$

$x = 300$

conf. min

$F'' = \frac{2(180{,}000)}{x^3}$

+ ∪

(x, y)

Groups: Find the points on the graph of $y = 4 - x^2$ that are closest to the point (0, 2).

$x^2 = 4 - y$

Minimizing distance:
so primary equation is the distance formula!

$d = \sqrt{(x-0)^2 + (y-2)^2}$

Secondary relationship is given.

$d = \sqrt{4 - y + y^2 - 4y + 4}$ ∴

$d = \sqrt{y^2 - 5y + 8}$

Let $g(y) = y^2 - 5y + 8$

$g'(y) = 2y - 5$

$0 = 2y - 5$

$y = \frac{5}{2}$

Confirm minimum

$g''(y) = 2$ always +

so concave up

Confirms min.

$\left(\quad, \frac{5}{2}\right)$ ≠ $\left(\quad, \frac{5}{2}\right)$

HW:

p. 197 # 13, 23, 27

p. 203 # 6, 15, 21, 23

Short Quiz tomorrow:
Analyze and accurately sketch a graph.